

# Absence of electron dephasing at zero temperature

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Dephasing of electrons due to the electron-electron interaction has recently been the subject of a controversial debate, with different calculations yielding mutually incompatible results. In this paper we prove, by means of Ward identities, that in a system with time reversal invariance, neither a Coulomb interaction nor a short-ranged model interaction can lead to phase breaking at zero temperature in spatial dimensions  $d > 2$ .

## I. INTRODUCTION

Many interesting phenomena in electronic systems at low temperatures depend on the electrons' phase coherence. Among these are the 'weak-localization effects',<sup>1</sup> which are nonanalytic dependences of zero-temperature ( $T = 0$ ) electronic correlation functions on frequency and/or wavenumber. Loss of phase coherence cuts off these singularities, and transfers the nonanalyticities into nonanalytic dependences on the phase breaking parameter. The standard interpretation of the weak-localization experiments is based on the temperature dependence of various phase breaking processes and the resulting non-analytic temperature dependence of various observables,<sup>2</sup> and an understanding of phase breaking processes is therefore of great importance. Of particular interest is phase breaking due to inelastic scattering caused by the electron-electron interaction, since at low temperatures this dominates other temperature dependent scattering processes, e.g., those caused by phonons.

For many years there had been consensus that at zero temperature the phase-breaking rate due to interactions vanishes, so that in the absence of other dephasing processes like, e.g., magnetic scattering, the electrons maintain full interference capabilities. This consensus was based on calculations of various phase-breaking times. An important example is the single-particle inelastic life time  $\tau_{\text{in}}$ , which describes the decay of the single-particle Green function,

$$G(\mathbf{p}, i\omega_n) = \frac{Z^{1/2}}{-i\omega_n + \xi_{\mathbf{p}} + i \operatorname{sgn}(\omega_n)/2\tau_{\text{in}}} \quad (1.1)$$

Here  $\mathbf{p}$  is the wavenumber,  $\xi_{\mathbf{p}} = \epsilon(\mathbf{p}) - \mu$  with  $\epsilon(\mathbf{p})$  the renormalized single-particle energy and  $\mu$  the chemical potential,  $\omega_n = 2\pi T(n + 1/2)$  is a fermionic Matsubara frequency, and  $Z$  is the wavefunction renormalization. Landau Fermi-liquid theory predicts the single-particle

inelastic scattering rate,  $1/\tau_{\text{in}}$ , in clean electron fluids to vanish like  $1/\tau_{\text{in}} \propto T^2$ .<sup>3</sup> Schmid calculated  $1/\tau_{\text{in}}$  for disordered systems with diffusive electron dynamics, and found it to be enhanced compared to the clean case, vanishing like  $T^{d/2}$  in  $2 < d < 4$  spatial dimensions.<sup>4</sup> The transport properties measured in weak-localization experiments are governed by a similar relaxation time, namely, the phase breaking time  $\tau_{\phi}$  that is related to a two-particle correlation function often referred to as the diffuson propagator,<sup>5,6</sup>

$$L(\mathbf{p}, i\Omega_n) = \frac{Z}{\Omega_n \tilde{H} + D\mathbf{p}^2 + 1/\tau_{\phi}} \quad (1.2)$$

Here  $\Omega_n = 2\pi Tn > 0$  is a bosonic Matsubara frequency,  $\tilde{H}$  is a frequency renormalization, and  $D$  is a renormalized diffusion coefficient. For  $2 < d < 4$ , many different calculation found the temperature dependences of  $\tau_{\phi}$  and  $\tau_{\text{in}}$  to be qualitatively the same,  $1/\tau_{\phi} \propto 1/\tau_{\text{in}} \propto T^{d/2}$ .<sup>7</sup> In a weak-localization context, instead of the diffuson, the closely related Cooperon propagator is often discussed, which has a mass that in time-reversal invariant systems is also given by  $\tau_{\phi}$ .<sup>8</sup> An overview within a unifying theoretical framework has been given in Ref. 9.

This consensus was shattered in the wake of experiments on disordered metallic wires that found that  $1/\tau_{\phi}$  saturates as  $T \rightarrow 0$ .<sup>10,11</sup> The experimentalists assert to have ruled out other sources of dephasing, which suggests that the electron-electron interaction can lead to residual dephasing even at  $T = 0$ . These experiments deal with finite-size one-dimensional or two-dimensional samples only, and for these systems an explanation in terms of quantum fluctuations has been proposed.<sup>12</sup> However, more detailed theoretical work along the same lines found such a residual contribution to  $1/\tau_{\phi}$  not just in  $d = 1$  and  $d = 2$ , but also in  $d = 3$ ,<sup>13</sup> and all of these results are in contradiction to the earlier theoretical work.<sup>8</sup> The quantum fluctuations that are proposed to lead to the satu-

ration have been taken into account in the earlier calculations, as Ref. 9 makes explicit, yet the results disagree. This contradiction has recently given rise to an intense debate.<sup>14,15</sup> It is important to note that this disagreement by far transcends the technical issue of what the behavior of a particular observable is, and how to calculate it. A finite dephasing rate at  $T = 0$  in three-dimensions would mean the breakdown of a Fermi-liquid picture for disordered bulk Fermi systems, with far-reaching consequences.

In this paper we attack the problem from a very general theoretical angle. Rather than calculating the phase-breaking rate explicitly, we use general symmetry arguments and Ward identities to prove that the electron-electron interaction cannot give rise to a nonvanishing value of  $1/\tau_\phi$  at zero temperature in  $d > 2$ . This mathematical result establishes that the diffuson propagator is always massless at zero temperature. We then show that in the presence of time reversal invariance, the same is true for the Cooperon propagator.

## II. SPONTANEOUS SYMMETRY BREAKING AND SOFT MODES IN ELECTRON SYSTEMS

The discussion in the literature of conflicting results obtained by different calculations that we summarized in the Introduction fails to mention an important point. Namely, the question of the existence or otherwise of a finite electron dephasing time at zero temperature is a question about the soft-mode spectrum of the system, since a nonzero dephasing rate gives certain propagators a mass, see Eq. (1.2). The existence and identity of soft modes, however, is governed by fundamental properties of the system and can be studied rigorously, and in very general terms, without reference to any particular explicit calculation. In fact, such an analysis should always be performed *before* carrying out calculations, since the former provides important checks for the latter. This is particularly important if the calculations involve approximations, as it is crucial to make sure that the approximations are ‘conserving’ in the sense that they do not violate any exact qualitative properties. Of course, systematic perturbation theory, if done correctly, is always conserving in this sense, as any exact properties will hold order by order in perturbation theory. However, given the complexity of many perturbative calculations, and the fact that perturbation theory by itself can never establish that some property is actually exact, it is still valuable to have exact benchmarks.

There are two known mechanisms for guaranteeing that certain correlation functions are massless, viz., (1) conservation laws, and (2) spontaneously broken symmetries. Conservation laws ensure that a conserved quantity cannot be locally created or destroyed. An excitation that couples to a conserved quantity can only decay by some of the quantity being transported into or out of the

perturbed region. For long-wavelength excitations, this takes a long time. Therefore, the corresponding correlation functions will decay infinitely slowly in the limit of zero wavenumber and zero frequency, i.e., they are massless. An example are density fluctuations in any isolated many-particle system, whose decay rate goes to zero with vanishing wavenumber due to the particle number conservation law. This is of course true both at zero and at nonzero temperature. A spontaneously broken continuous symmetry also leads to soft modes, the number of which depends on the number of parameters of the symmetry group that is broken, and on the extent of the breaking. This is the content of Goldstone’s theorem, and the resulting soft modes are usually referred to as Goldstone modes.<sup>16</sup> An example are the spin waves in a Heisenberg antiferromagnet, which are the Goldstone modes resulting from the spontaneously broken spin rotation symmetry.

The systematic analysis of symmetries and the resulting soft modes in disordered itinerant electron systems was pioneered by Wegner,<sup>17</sup> and by McKane and Stone.<sup>18</sup> These authors showed that, for noninteracting electrons, the diffusive excitations that give rise to the weak-localization effects are the Goldstone modes of a spontaneously broken symmetry that can be formulated as a rotation in frequency space. Roughly speaking, it is the symmetry between retarded and advanced Green functions that is spontaneously broken whenever there is a nonzero density of states. A nonzero temperature serves as an external field conjugate to the order parameter, i.e. the density of states, and gives the Goldstone modes a mass. The results of Refs. 13, 14 amount to the claim that either, the electron-electron interaction provides such a conjugate field even at  $T = 0$ , making the Goldstone modes massive, or, that time reversal invariance is always broken in interacting many-body systems, making the Cooperon different from the diffuson.

Wegner’s symmetry analysis has been generalized to the case of interacting electrons in Refs. 19, 20, and was used to construct an effective field theory for disordered interacting electrons. Although it is implicit in this work that at zero temperature, and in  $d > 2$ , the electron-electron interaction cannot lead to any dephasing, the controversy discussed above makes it worthwhile to re-analyze the symmetry arguments and prove this statement explicitly. We will now proceed to do so.

## III. ABSENCE OF PHASE BREAKING AT ZERO TEMPERATURE

### A. Field theory for fermions

Our starting point is a general field theory for electrons. For any fermionic system, the partition function can be written<sup>21</sup>

$$Z = \int D[\bar{\psi}, \psi] \exp(S[\bar{\psi}, \psi]) \quad , \quad (3.1a)$$

where  $S$  is the action in terms of the fermionic (i.e., Grassmann valued) fields  $\bar{\psi}$  and  $\psi$ . We consider an action that consists of a free-fermion part  $S_0$ , a part  $S_{\text{dis}}$  describing the interaction of the electrons with quenched disorder, and a part  $S_{\text{int}}$  describing the electron-electron interaction,

$$S = S_0 + S_{\text{dis}} + S_{\text{int}} \quad , \quad (3.1b)$$

Each field  $\psi$  or  $\bar{\psi}$  carries a Matsubara frequency index  $n$  and a spin index  $\sigma = \uparrow, \downarrow$ . In terms of these fields,  $S_0$  reads explicitly

$$S_0 = - \int d\mathbf{x} \sum_{n, \sigma} \bar{\psi}_{n\sigma}(\mathbf{x}) [i\omega_n + \epsilon(\nabla) + \mu] \psi_{n\sigma}(\mathbf{x}) \quad , \quad (3.2)$$

with  $\mu$  the chemical potential, and  $\omega_n = 2\pi T(n + 1/2)$  a fermionic Matsubara frequency.  $\epsilon$  denotes the dispersion relation. For instance, for free electrons one has  $\epsilon(\nabla) = -\nabla^2/2m$ , with  $m$  the electron mass. Since we will be interested in four-fermion correlation functions, it is useful to introduce a matrix of bilinear products of the fermion fields,<sup>22</sup>

$$B_{nm} = \frac{i}{2} \begin{pmatrix} -\psi_{n\uparrow}\bar{\psi}_{m\uparrow} & -\psi_{n\uparrow}\bar{\psi}_{m\downarrow} & -\psi_{n\downarrow}\bar{\psi}_{m\downarrow} & \psi_{n\downarrow}\bar{\psi}_{m\uparrow} \\ -\psi_{n\downarrow}\bar{\psi}_{m\uparrow} & -\psi_{n\downarrow}\bar{\psi}_{m\downarrow} & -\psi_{n\uparrow}\bar{\psi}_{m\downarrow} & \psi_{n\uparrow}\bar{\psi}_{m\uparrow} \\ \psi_{n\downarrow}\bar{\psi}_{m\uparrow} & \psi_{n\downarrow}\bar{\psi}_{m\downarrow} & \psi_{n\uparrow}\bar{\psi}_{m\downarrow} & -\psi_{n\uparrow}\bar{\psi}_{m\uparrow} \\ -\psi_{n\uparrow}\bar{\psi}_{m\uparrow} & -\psi_{n\uparrow}\bar{\psi}_{m\downarrow} & -\psi_{n\downarrow}\bar{\psi}_{m\downarrow} & \psi_{n\downarrow}\bar{\psi}_{m\uparrow} \end{pmatrix} \\ \cong Q_{nm} \quad . \quad (3.3)$$

where all fields are understood to be taken at position  $\mathbf{x}$ . The matrix elements of  $B$  commute with one another, and are therefore isomorphic to classical or number-valued fields that we denote by  $Q$ .<sup>23</sup> This isomorphism maps the adjoint operation on products of fermion fields, which is denoted above by an overbar, on the complex conjugation of the classical fields. We use the isomorphism to constrain  $B$  to the classical field  $Q$ , and exactly rewrite the partition function<sup>19</sup>

$$Z = \int D[\bar{\psi}, \psi] e^{S[\bar{\psi}, \psi]} \int D[Q] \delta[Q - B] \\ = \int D[\bar{\psi}, \psi] e^{S[\bar{\psi}, \psi]} \int D[Q] D[\tilde{\Lambda}] e^{\text{Tr}[\tilde{\Lambda}(Q - B)]} \\ \equiv \int D[Q] D[\tilde{\Lambda}] e^{\mathcal{A}[Q, \tilde{\Lambda}]} \quad . \quad (3.4)$$

$\tilde{\Lambda}$  is an auxiliary bosonic matrix field that serves to enforce the functional delta-constraint in the first line of Eq. (3.4), and the last line defines the action  $\mathcal{A}$ . The matrix elements of both  $Q$  and  $\tilde{\Lambda}$  are spin-quaternions (i.e., elements of  $\mathcal{Q} \times \mathcal{Q}$  with  $\mathcal{Q}$  the quaternion field). From Eq. (3.3) we see that expectation values of the

$Q$  matrix elements yield local Green functions, and  $Q$ - $Q$  correlation functions describe four-fermion correlation functions. The physical meaning of  $\tilde{\Lambda}$  is that its expectation value plays the role of a self energy, see Ref. 19.

It is convenient to expand the  $4 \times 4$  matrix in Eq. (3.3) in a spin-quaternion basis,

$$Q_{nm}(\mathbf{x}) = \sum_{r, i=0,3} (\tau_r \otimes s_i)^i Q_{nm}(\mathbf{x}) \quad (3.5)$$

and analogously for  $\tilde{\Lambda}$ . Here  $\tau_0 = s_0 = \mathbb{1}_2$  is the  $2 \times 2$  unit matrix, and  $\tau_j = -s_j = -i\sigma_j$ , ( $j = 1, 2, 3$ ), with  $\sigma_{1,2,3}$  the Pauli matrices. In this basis,  $i = 0$  and  $i = 1, 2, 3$  describe the spin singlet and the spin triplet, respectively. An explicit calculation reveals that  $r = 0, 3$  corresponds to the particle-hole channel (i.e., products  $\bar{\psi}\psi$ ), while  $r = 1, 2$  describes the particle-particle channel (i.e., products  $\bar{\psi}\bar{\psi}$  or  $\psi\psi$ ). We will be particularly interested in the matrix elements of  ${}_0^0Q$  and  ${}_1^0Q$ , for which the isomorphism expressed in Eq. (3.3) reads

$${}_0^0Q_{12}(\mathbf{x}) \cong \frac{i}{8} \sum_{\sigma} [\bar{\psi}_{1\sigma}(\mathbf{x})\psi_{2\sigma}(\mathbf{x}) + \bar{\psi}_{2\sigma}(\mathbf{x})\psi_{1\sigma}(\mathbf{x})] \quad , \quad (3.6a)$$

$${}_1^0Q_{12}(\mathbf{x}) \cong \frac{1}{8} [\bar{\psi}_{1\uparrow}(\mathbf{x})\bar{\psi}_{2\downarrow}(\mathbf{x}) + \bar{\psi}_{2\uparrow}(\mathbf{x})\bar{\psi}_{1\downarrow}(\mathbf{x}) \\ + \psi_{1\uparrow}(\mathbf{x})\psi_{2\downarrow}(\mathbf{x}) + \psi_{2\uparrow}(\mathbf{x})\psi_{1\downarrow}(\mathbf{x})] \quad . \quad (3.6b)$$

From the structure of Eq. (3.3) one obtains the following formal symmetry properties of the  $Q$  matrices,<sup>19,20</sup>

$${}_r^0Q_{nm} = (-)^r {}_r^0Q_{mn} \quad , \quad (r = 0, 3) \quad , \quad (3.7a)$$

$${}_i^rQ_{nm} = (-)^{r+1} {}_i^rQ_{mn} \quad , \quad (r = 0, 3; i = 1, 2, 3) \quad , \quad (3.7b)$$

$${}_r^0Q_{nm} = {}_r^0Q_{mn} \quad , \quad (r = 1, 2) \quad , \quad (3.7c)$$

$${}_i^rQ_{nm} = -{}_i^rQ_{mn} \quad , \quad (r = 1, 2; i = 1, 2, 3) \quad , \quad (3.7d)$$

$${}_i^rQ_{nm}^* = -{}_i^rQ_{-n-1, -m-1} \quad . \quad (3.7e)$$

Here the star in Eq. (3.7e) denotes complex conjugation.

By using the delta constraint in Eq. (3.4) to rewrite all terms that are quartic in the fermion field in terms of  $Q$ , we can achieve an integrand that is bilinear in  $\psi$  and  $\bar{\psi}$ . The Grassmannian integral can then be performed exactly, and we obtain for the action  $\mathcal{A}$

$$\mathcal{A}[Q, \tilde{\Lambda}] = \mathcal{A}_{\text{dis}} + \mathcal{A}_{\text{int}} + \frac{1}{2} \text{Tr} \ln (G_0^{-1} - i\tilde{\Lambda}) \\ + \int d\mathbf{x} \text{tr} (\tilde{\Lambda}(\mathbf{x}) Q(\mathbf{x})) \quad . \quad (3.8a)$$

Here

$$G_0^{-1} = -\partial_{\tau} + \epsilon(\nabla) + \mu \quad , \quad (3.8b)$$

is the inverse free electron Green operator, with  $\partial_{\tau}$  the derivative with respect to imaginary time.  $\text{Tr}$  denotes a

trace over all degrees of freedom, including the continuous position variable, while  $\text{tr}$  is a trace over all those discrete indices that are not explicitly shown.

For explicit calculations, the electron-electron interaction  $\mathcal{A}_{\text{int}}$  is usually decomposed into four pieces that describe the interaction in the particle-hole and particle-particle spin-singlet and spin-triplet channels, respectively.<sup>24,19</sup> For the purposes of the present paper, which will study exact structural properties rather than explicit calculations, this decomposition is neither necessary nor desirable. We therefore write the interaction part of the action as the basic density-density interaction between electrons mediated by a statically screened Coulomb potential. In terms of fermionic fields, this reads

$$S_{\text{int}} = -\frac{1}{2} \int d\mathbf{x} d\mathbf{y} v_{\text{sc}}(\mathbf{x} - \mathbf{y}) \sum_{\sigma_1, \sigma_2} \bar{\psi}_{\sigma_1}(\mathbf{x}) \psi_{\sigma_1}(\mathbf{x}) \times \bar{\psi}_{\sigma_2}(\mathbf{y}) \psi_{\sigma_2}(\mathbf{y}) \quad . \quad (3.9a)$$

Here

$$v_{\text{sc}}(\mathbf{x}) = \frac{e^2}{|\mathbf{x}|} e^{-\kappa|\mathbf{x}|} \quad , \quad (3.9b)$$

is the screened Coulomb potential, with  $e$  the electron charge and  $\kappa$  the Thomas-Fermi screening wavenumber. We stress that we use a screened interaction for convenience (and since screening is a real physical effect) only, and that our arguments do not rely on this. A suitable modification of the procedure in Secs. III B and III C still applies if one works with a bare Coulomb interaction, and the results are the same, see Ref. 20.

Its Fourier transform is, in  $d$  spatial dimensions,<sup>25</sup>

$$v_{\text{sc}}(\mathbf{q}) = \frac{1}{N_F} \frac{\kappa^{(d-1)}}{[\mathbf{q}^2 + \kappa^2]^{(d-1)/2}} \quad , \quad (3.9c)$$

with  $N_F$  the density of states of clean, noninteracting electrons at the Fermi level. In terms of the  $Q$  matrices, Eq. (3.9a) reads

$$\mathcal{A}_{\text{int}} = 8T \int d\mathbf{x} d\mathbf{y} \sum_{r=0,3} \sum_{n_1, n_2, m} v_{\text{sc}}(\mathbf{x} - \mathbf{y}) \times {}^0_r Q_{n_1, n_1+m}(\mathbf{x}) {}^0_r Q_{n_2, n_2+m}(\mathbf{y}) \quad , \quad (3.9d)$$

Finally, the interaction of the electrons with the static disorder is given by a random potential  $u(\mathbf{x})$  that couples to the electronic density,

$$S_{\text{dis}} = - \int d\mathbf{x} u(\mathbf{x}) \sum_{\sigma} \sum_n \bar{\psi}_{n, \sigma}(\mathbf{x}) \psi_{n, \sigma}(\mathbf{x}) \quad , \quad (3.10a)$$

or, in terms of the  $Q$ ,

$$\mathcal{A}_{\text{dis}} = 4i \int d\mathbf{x} u(\mathbf{x}) \sum_n {}^0_r Q_{nn}(\mathbf{x}) \quad . \quad (3.10b)$$

The statistical properties of the random function  $u(\mathbf{x})$  are governed by a distribution functional  $P[u]$ . The partition function, Eq. (3.4), is a functional of  $u(\mathbf{x})$ , and any physical quantity  $X$ , like the free energy, or any correlation function, needs to be averaged over the disorder according to

$$\{X\}_{\text{dis}} = \int D[u(\mathbf{x})] X[u] P[u] \quad . \quad (3.10c)$$

In Refs. 19, 20 the disorder average was performed by means of the replica trick. Here, since we are not concerned about explicit calculations, we will formally perform the disorder average by means of the basic definition, Eq. (3.10c).

The above definitions completely specify our model. We stress that the  $Q$ -matrices merely provide a convenient shorthand for bilinear products of fermion fields. It would be perfectly possible to proceed in terms of the latter, and whether or not one introduces the matrix fields is a matter of taste.

## B. Symmetry Considerations, and a Ward identity

A symmetry analysis of the action, Eqs. (3.8) - (3.10), has been given in Refs. 19, 20. Here we recapitulate those aspects of this analysis that are important for our purposes.

Let us consider an infinitesimal transformation of the  $Q$ -matrices,  $Q \rightarrow TQT^{-1}$ , with  $T = C\tilde{T}C^T$ , where  $C = i\tau_1 \otimes s_2$ , and

$$\hat{T}_{nm} = (\delta_{nn_1} \delta_{mn_2} - \delta_{nn_2} \delta_{mn_1}) \theta + O(\theta^2) \quad . \quad (3.11)$$

$n_1 > 0$  and  $n_2 < 0$  are fixed frequency indices that characterize the transformation. This amounts to a rotation in frequency space by an infinitesimal angle  $\theta$ . These rotations are elements of the symplectic symmetry group  $\text{Sp}(8N)$  over the complex numbers, with  $2N$  the number of frequency labels, which governs the symmetry properties of the action.<sup>19</sup> Under such a rotation, the  $Q$ -matrices transform like

$$Q_{nm} \rightarrow Q_{nm} + \delta Q_{nm} \quad , \quad (3.12a)$$

with

$$\delta Q_{nm} = (\delta_{nn_1} Q_{n_2m} + \delta_{mn_1} Q_{nn_2} - (1 \leftrightarrow 2)) \theta + O(\theta^2) \quad . \quad (3.12b)$$

The  $\tilde{\Lambda}$  transform analogously. The symbol  $(1 \leftrightarrow 2)$  denotes the same terms as written previously, but with the indices 1 and 2 interchanged.

Of the terms in the action, Eq. (3.8a), only  $\mathcal{A}_{\text{int}}$  and the  $\text{Tr} \ln$  term are not invariant under these rotations. To linear order in the transformation parameter  $\theta$  one finds  $\mathcal{A} \rightarrow \mathcal{A} + \delta\mathcal{A}$ , with

$$\delta\mathcal{A} = \frac{\theta}{2} \text{Tr} (G \delta i\omega) + \delta\mathcal{A}_{\text{int}} \quad , \quad (3.13a)$$

with  $G \equiv (G_0^{-1} - i\tilde{\Lambda})^{-1}$ , and

$$(\delta i\omega)_{nm}^{\alpha\beta} = (\delta_{nn_1}\delta_{mn_2} + \delta_{nn_2}\delta_{mn_1}) i\Omega_{n_1-n_2} \quad , \quad (3.13b)$$

$$\delta\mathcal{A}_{\text{int}} = 32 \int d\mathbf{x} d\mathbf{y} v_{\text{sc}}(\mathbf{x} - \mathbf{y}) \sum_{r=0,3} T \sum_{n_a n_b} \left[ {}^0_r Q_{n_a n_b}(\mathbf{x}) \right. \\ \left. \times {}^0_r Q_{n_2, n_2-(n_a-n_b)}(\mathbf{y}) - (1 \leftrightarrow 2) \right] \theta \quad . \quad (3.13c)$$

Now introduce a source  $J$  for the  $Q$ -fields, and consider the generating functional

$$Z[J] = \int D[Q] D[\tilde{\Lambda}] e^{\mathcal{A} + \int d\mathbf{x} \text{tr} (J(\mathbf{x}) Q(\mathbf{x}))} \quad . \quad (3.14)$$

By performing an infinitesimal transformation on the  $Q$  and the  $\tilde{\Lambda}$ , one obtains from Eq. (3.14)

$$0 = \int D[Q] D[\tilde{\Lambda}] \left[ \delta\mathcal{A} + \int d\mathbf{x} \text{tr} (J(\mathbf{x}) \delta Q(\mathbf{x})) \right] \\ \times e^{\mathcal{A} + \int d\mathbf{x} \text{tr} (J(\mathbf{x}) Q(\mathbf{x}))} \quad . \quad (3.15a)$$

Differentiating this identity with respect to  ${}^0_{n_4 n_3}$ ,  $n_3 > 0$ ,  $n_4 < 0$ , and putting  $J = 0$  yields

$$0 = \left\langle \delta\mathcal{A} {}^0_{n_1 n_2} Q(\mathbf{x}) \right\rangle + \left\langle {}^0_0 (\delta Q)_{n_1 n_2}(\mathbf{x}) \right\rangle \quad , \quad (3.15b)$$

Here the angular brackets denote an average with respect to the action  $\mathcal{A}$ . From Eq. (3.13a) we see that the first term involves an average  $\langle GQ \rangle$ . By using the identity<sup>19</sup>  $\langle GQ \rangle = -2i\langle Q^2 \rangle$ , and performing the disorder average, Eq. (3.10c), we finally obtain

$$8\Omega_{n_1-n_2} D_{n_1 n_2, n_3 n_4} = \delta_{n_1 n_3} \delta_{n_2 n_4} N_{n_1 n_2} - W_{n_1 n_2, n_3 n_4} \quad . \quad (3.16a)$$

Here

$$N_{n_1 n_2} = \left\{ \left\langle {}^0_{n_1 n_1} Q(\mathbf{x}) \right\rangle - \left\langle {}^0_{n_2 n_2} Q(\mathbf{x}) \right\rangle \right\}_{\text{dis}} \quad , \quad (3.16b)$$

$$D_{n_1 n_2, n_3 n_4} = \int d\mathbf{y} \left\{ \left\langle {}^0_0 (\Delta Q)_{n_1 n_2}(\mathbf{y}) \right. \right. \\ \left. \left. \times {}^0_0 (\Delta Q)_{n_3 n_4}(\mathbf{x}) \right\rangle \right\}_{\text{dis}} \quad , \quad (3.16c)$$

with  $\Delta Q = Q - \langle Q \rangle$ , and

$$W_{n_1 n_2, n_3 n_4} = -32 v_{\text{sc}}(\mathbf{k} \rightarrow 0) N_{n_1 n_2} T \sum_{n_a, n_b} \delta_{n_a - n_b, n_1 - n_2} \\ \times D_{n_a n_b, n_3 n_4} \\ - 32 \sum_{r=0,3} \int d\mathbf{y} d\mathbf{z} v_{\text{sc}}(\mathbf{y} - \mathbf{z}) T \sum_{n_a, n_b} \\ \times \left[ \left\{ \left\langle {}^0_r (\Delta Q)_{n_1, n_2-(n_a-n_b)}(-\mathbf{q}) {}^0_r (\Delta Q)_{n_a n_b}(\mathbf{q}) \right. \right. \right. \\ \left. \left. \times {}^0_0 (\Delta Q)_{n_3 n_4}(\mathbf{x}) \right\rangle \right\}_{\text{dis}} - (1 \leftrightarrow 2) \right] \quad . \quad (3.16d)$$

The physical meaning of the propagator  $D$  is a phase space density correlation function. The homogeneous, static density susceptibility  $\chi$  can be obtained from  $D$  by the relation<sup>19</sup>  $\chi = 16T \sum_{n,m} D_{nn,mm}$ .

The Ward identity,<sup>19,20</sup> Eq. (3.16a), relates the two-point  $Q$ -correlation function  $D$  to the one-point function  $N$  and the three-point function  $W$ . Various aspects of this identity have been discussed in Refs. 19, 20. In the next subsection we focus on the implications for the dephasing time.

### C. Absence of Dephasing

We now use the Ward identity, Eq. (3.16a), to show that there cannot be a nonzero phase relaxation rate at zero temperature in  $d > 2$ . We will first concentrate on the diffuson propagator, and then show that, in time reversal invariant systems, the same conclusions hold for the Cooperon propagator.

#### 1. The diffuson propagator

Let us consider the structure of the Ward identity, Eq. (3.16a). Equation (3.6a) shows that  $\langle {}^0_{nn} Q(\mathbf{x}) \rangle$  is proportional to the on-site electron Green function, so for  $n_1 > 0$ ,  $n_2 < 0$ ,  $\Omega_{n_1-n_2} \equiv \omega_{n_1} - \omega_{n_2} \rightarrow 0$ ,  $N_{n_1 n_2}$  approaches a constant  $N$  that is proportional to the density of states at the Fermi level.  $D$  and  $W$  can be decomposed into two pieces each that have different frequency structures,

$$D_{n_1 n_2, n_3 n_4} = \delta_{n_1, n_3} \delta_{n_2, n_4} D_{n_1 n_2}^{(\text{dc})} \\ + \delta_{n_1 - n_2, n_3 - n_4} D_{n_1 n_2, n_3 n_4}^{(\text{c})} \quad , \quad (3.17)$$

and analogously for  $W$ . The superscripts ‘(c)’ and ‘(dc)’ refer to ‘connected’ and ‘disconnected’ (at a fermionic level) contributions. The disconnected piece of  $D$ ,  $D^{(\text{dc})}$ , is identical, apart from a proportionality constant, with the homogeneous limit ( $\mathbf{p} \rightarrow 0$ ) of the diffuson propagator  $L$ , Eq. (1.2), that was defined diagrammatically in Ref. 5. For noninteracting electrons, the connected piece of  $D$  vanishes, and we have, in the limit  $\Omega_{n_1-n_2} \rightarrow 0$ ,

$$D_{n_1 n_2}^{(\text{dc})} \rightarrow N/8\Omega_{n_1-n_2} \quad . \quad (3.18)$$

This expresses the undisputed fact that in the absence of an electron-electron interaction, the diffuson is massless at zero temperature.<sup>26</sup>

The question is now whether the presence of  $W$  can change this fact. The only possibility is a cancellation between  $N$  and  $W$  in the limit of zero frequency and zero temperature. To see that this cannot happen, let us consider the equation for  $D^{(\text{dc})}$  in this case. It is sufficient to show that  $D^{(\text{dc})}$  remains massless, since  $D^{(\text{c})}$  and  $D^{(\text{dc})}$  cannot cancel one another due to their different frequency structures. From Eq. (3.16a) we obtain

$$8\Omega_{n_1-n_2} D_{n_1 n_2}^{(\text{dc})} = N - W^{(\text{dc})} \quad , \quad (3.19a)$$

An evaluation of Eq. (3.16d) yields

$$W^{(\text{dc})} = 8 \lim_{\substack{\omega_{n_1} \rightarrow 0+ \\ \omega_{n_2} \rightarrow 0-}} \int d\mathbf{y} d\mathbf{z} v_{\text{sc}}(\mathbf{y} - \mathbf{z}) T \sum_{n_a, n_b} \times \left\{ G(\mathbf{x}, \mathbf{z}; \omega_{n_1}) \left\langle (\bar{\psi}_{n_a}(\mathbf{y}) s_0 \psi_{n_b}(\mathbf{y})) \right. \right. \\ \left. \left. \times ((\bar{\psi}_{n_2-n_a+n_b}(\mathbf{z}) s_0 \psi_{n_2}(\mathbf{x}))) \right\rangle^c - (1 \leftrightarrow 2) \right\}_{\text{dis}} . \quad (3.19b)$$

Here we have introduced two-component spinors,  $\psi_n(\mathbf{x}) = (\psi_{n\uparrow}(\mathbf{x}), \psi_{n\downarrow}(\mathbf{x}))$ , and a scalar product in spinor space that is given by the usual matrix product. The  $\langle \bar{\psi} \psi \bar{\psi} \psi \rangle^c$  are connected  $\psi$ -correlation functions, with the cumulant taken with respect to the quantum mechanical average only, and  $G(\mathbf{x}, \mathbf{z}; i\omega_n) = \langle \bar{\psi}_{n\sigma}(\mathbf{x}) \psi_{n\sigma}(\mathbf{z}) \rangle$  is the Green function for a given disorder configuration.

Now consider  $W^{(\text{dc})}$ , Eq. (3.19b), as a function of the screening wavenumber  $\kappa$ . The behavior for small  $\kappa$  is determined by the infrared properties of the integral that defines  $W^{(\text{dc})}$ . The connected four-fermion correlation functions contain at least one screened interaction potential, and any four-fermion function is at most diffusive, i.e. diverges at most like an inverse wavenumber squared for small wavenumbers. Counting the number of frequency and momentum integrals in Eq. (3.19b), we conclude that the leading zero-temperature contribution to  $W^{(\text{dc})}$  for  $\kappa \rightarrow 0$  is bounded above by

$$W^{(\text{dc})} < \text{const.} \int_0^\Lambda dp p^{d-1} \frac{\kappa^{2(d-1)}}{[\kappa^2 + p^2]^{d-1}} \frac{1}{p^2} \\ \propto \kappa^{d-2} + o(\kappa^{d-2}) \quad , \quad (3.20)$$

where  $o(x)$  denotes terms that vanish faster than  $x$  as  $x \rightarrow 0$ .  $\Lambda$  is an ultraviolet cutoff that is on the order of the Fermi wavenumber. It follows that  $W^{(\text{dc})}$  vanishes continuously as  $\kappa \rightarrow 0$  for  $d > 2$ . The density of states, in contrast, approaches a finite value, and so  $N = O(1)$  in this limit. We see that  $N$  and  $W^{(\text{dc})}$  are different functions of  $\kappa$ , so in general they cannot cancel. This means that the diffuson propagator remains soft in the presence of interactions. It can become massive at most at a special value of the screening wavenumber, which would signalize a phase transition into a non-Fermi liquid state. If one replaces the screened Coulomb interaction by a different short-ranged model interaction with interaction amplitude  $\Gamma$ ,<sup>27</sup> then all of the above arguments still apply if one considers  $N$  and  $W^{(\text{dc})}$  as functions of  $\Gamma$  instead of  $\kappa$ .

The realization that  $W$  and  $N$  are necessarily different functions of the interaction, and therefore cannot generically cancel, is the most important ingredient of our proof. Notice that a cancellation would require  $W^{(\text{dc})}$  to be of  $O(1)$  for  $\kappa \rightarrow 0$ . It follows from Eq. (3.20) that the

only way for this to happen is to have a stronger than diffusive infrared singularity in a four-fermion propagator. However, the nonvanishing phase relaxation rate that might result from this would make the diffusive Goldstone modes massive, i.e., it would lead to a theory that is *less* singular in the infrared. Any scenario that has the diffuson become massive due to the electron-electron interaction would therefore not even be internally consistent.

We finally note that no cancellation between  $N$  and  $W^{(\text{dc})}$  is necessary to produce dephasing at a nonzero temperature. According to the relation between  $D^{(\text{dc})}$  and  $L$  from Eq. (1.2) mentioned above, and after an analytic continuation to real frequencies  $\Omega$ , one has

$$N - W^{(\text{dc})} = \frac{\text{const.}}{\tilde{H} + i/\Omega\tau_\phi} \quad . \quad (3.21)$$

As long as, for  $T \rightarrow 0$  and  $\Omega \rightarrow 0$ , a regime exists where  $\Omega\tau_\phi \gg 1$ , Eq. (3.21) allows for an expansion in powers of  $1/\Omega\tau_\phi$ , with the leading contribution to  $N - W^{(\text{dc})}$  a nonzero constant. This is the case if  $1/\tau_\phi$  vanishes continuously as  $T \rightarrow 0$ , and in particular if  $\tau_\phi \propto T^{d/2}$ . However, in order to have  $\tau_\phi \rightarrow \text{const.}$  for  $T \rightarrow 0$ , the leading  $O(1)$  contribution must cancel, which is generically impossible as shown above.

## 2. The Cooperon propagator

Finally, we return to the Cooperon propagator, which in our language is given by the correlation function

$$C_{n_1 n_2, n_3 n_4} = \int d\mathbf{y} \left\{ \left\langle \left\langle \begin{aligned} & \left\langle \bar{\psi}_1^0(\Delta Q)_{n_1 n_2}(\mathbf{y}) \right. \right. \\ & \left. \left. \times \bar{\psi}_1^0(\Delta Q)_{n_3 n_4}(\mathbf{x}) \right\rangle^0 \right\rangle_{\text{dis}} \right\} \quad , \quad (3.22)$$

The decomposition analogous to Eq. (3.17) reads in this case

$$C_{n_1 n_2, n_3 n_4} = \delta_{n_1, n_3} \delta_{n_2, n_4} C_{n_1 n_2}^{(\text{dc})} \\ + \delta_{n_1+n_2, n_3+n_4} C_{n_1 n_2, n_3 n_4}^{(c)} \quad , \quad (3.23)$$

From Eq. (3.6b) we see that the disconnected piece can be expressed in terms of Green functions as

$$C_{nm}^{(\text{dc})} = \frac{-1}{16} \sum_{\mathbf{k}, \mathbf{p}} \{ \mathcal{G}_{\mathbf{k}, \mathbf{p}}(i\omega_n) \mathcal{G}_{-\mathbf{k}, -\mathbf{p}}(i\omega_m) \}_{\text{dis}} \quad . \quad (3.24a)$$

Here  $\mathcal{G}$  is the electron Green function before disorder averaging. It is related to  $G$ , Eq. (1.1), by  $\{ \mathcal{G}_{\mathbf{k}, \mathbf{p}}(i\omega_n) \}_{\text{dis}} = \delta_{\mathbf{k}, \mathbf{p}} G(\mathbf{k}, i\omega_n)$ . Analogously, the diffuson propagator can be written

$$D_{nm}^{(\text{dc})} = \frac{1}{16} \sum_{\mathbf{k}, \mathbf{p}} \{ \mathcal{G}_{\mathbf{k}, \mathbf{p}}(i\omega_n) \mathcal{G}_{\mathbf{p}, \mathbf{k}}(i\omega_m) \}_{\text{dis}} \quad . \quad (3.24b)$$

In the presence of time reversal invariance one has the identity

$$\mathcal{G}_{\mathbf{k},\mathbf{p}}(i\omega_n) = \mathcal{G}_{-\mathbf{p},-\mathbf{k}}(i\omega_n) \quad . \quad (3.25)$$

Substituting this into Eq. (3.24a) leads to

$$C_{nm}^{(\text{dc})} = -D_{nm}^{(\text{dc})} \quad , \quad (3.26)$$

in time reversal invariant systems. This proves that the Cooperon is massless as well.

#### IV. CONCLUSION

In summary, we have considered a model of electrons in the presence of quenched disorder. For this model, we have proved that the diffuson propagator in  $d > 2$  at  $T = 0$  remains massless in the presence of an electron-electron interaction. This means that the dephasing rate due interactions vanishes at  $T = 0$ . In systems that are time-reversal invariant, the Cooperon propagator is also massless. These results prove that, to the extent that time reversal invariance holds for the systems studied in Ref. 10, the calculations and arguments given in Refs. 13, 14 cannot be used to explain the experimental observations.

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<sup>25</sup> Equation (3.9c) gives the exact Fourier transform for  $d = 2$  and  $d = 3$ , and an interpolation that has the correct scaling behavior for other values of  $d$ .

<sup>26</sup> In the absence of other dephasing mechanisms, this statement remains true at nonzero temperatures, where in the limit of a small real frequency  $\Omega$ ,  $D^{(\text{dc})} \rightarrow iN(T)/8\Omega$ .

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